International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

http://ijpsat.ijsht-journals.org
Vol. 17 No. 1 October 2019, pp. 31-33

Variational Principle for Traveling Waves in a Modified Kuramoto-Sivashinsky Equation

Erni Laswinda, Mahdhivan Syafwan*, Arrival Rince Putri

Department of Mathematics, Faculty of Mathematics and Natural Sciences,
Andalas University, Padang Indonesia

Abstract - In this paper, variational principle for traveling waves in a modified Kuramoto-Sivashinsky equation is constructed by the semi-inverse method. We confirm that the method is very effective and gives the results in the concise form of variational functionals.

Keywords - Kuramoto-Sivashinsky Equation, Variational Principle, Semi-Inverse Method.

I. Introduction

Variational principle for traveling waves in nonlinear partial differential equations plays an important role in physics and mathematics. Some researchers have obtained some variational principles by the Noether's theorem (see, e.g., [1]). However the derivation is rather heavy and tedious mathematically. Alternatively, He developed the so-called semi-inverse method [2] and has been widely used and claimed as a promising method to formulate the variational principles due to its simplicity and concise result (see, e.g., [3, 4, 5]).

In this paper, we will employ the semi-inverse method to establish variational principle for traveling waves in a modified Kuramoto-Sivashinsky equation as follows:

$$u_t + \frac{1}{2}u_x^2 + (\alpha u_x + 2\beta)u_{xx} + 2\gamma u_{xxx} + u_{xxxx} = 0. \qquad (1)$$

When $\alpha = \gamma = 0$ and $\beta = \frac{1}{2}$, Eq. (1) is called the Kuramoto-Sivashinsky equation in 'integral' form [6]. The equation was developed saparately by Yoshiki Kuramoto when studying the chaotic phenomena caused by diffusion process in the reaction system [7], and Gregory Sivashinsky when studying the flame front propagation [8]. Without loss of generality, here we consider the case $\alpha = \beta = \gamma = 1$.

II. Variational Principle by the Semi-Inverse Method

In order to seek traveling wave solutions of Eq. (1), we introduce the following transformation

$$u(x,t) = v(\xi), \quad \xi = kx + \lambda t, \qquad (2)$$

where k represents wave number and λ denotes wave velocity. For the sake of simplicity, let us set $k = \lambda = 1$. Substituting the transformations (2) into (1) with $\alpha = \beta = \gamma = k = \lambda = 1$ yields

$$v' + \frac{1}{2}(v')^2 + (v' + 2)v'' + 2v^{(3)} + v^{(4)} = 0, \qquad (3)$$

where the prime denotes the derivative with respect to ξ.

Let us first consider the following simple case

$$v'v'' + v'' + v^{(4)} = 0. \qquad (4)$$

One can check that

$$\tilde{L}_1 = -\frac{1}{6}(v')^3 \text{ for } v'v'' = 0,$$

$$\tilde{L}_2 = -\frac{1}{2}(v')^2 \text{ for } v'' = 0,$$

$$\tilde{L}_3 = \frac{1}{2}(v'')^2 \text{ for } v^{(4)} = 0.$$

From the above results, we can easily show that the variational principle for Eq. (4) is :

$$\tilde{L}(v) = \int -\frac{1}{2}(v')^2 - \frac{1}{6}(v')^3 + \frac{1}{2}(v'')^2 d\xi. \qquad (5)$$

Next, according to the semi-inverse method [2], we introduce an integrating factor $f(\xi)$, which is an unknown function of wave variable ξ, and consider the following integral :

$$\mathrm{L}(\mathrm{v}) = \int \mathrm{f}(\xi)\left[-\frac{1}{2}(\mathrm{v}')^2 - \frac{1}{6}(\mathrm{v}')^3 + \frac{1}{2}(\mathrm{v}'')^2\right] + \mathrm{F}(\mathrm{v}, \mathrm{v}', \mathrm{v}'', \dots)d\xi, \qquad (6)$$

where F is an unknown function of v and/or its derivatives. The Euler–Lagrange equation of Eq. (6) reads

$$f'v' + \frac{1}{2}f'(v')^2 + fv'v'' + (f + f'')v'' + 2f'v^{(3)} + fv^{(4)} + \frac{\delta F}{\delta v} = 0, \qquad (7)$$

where $\frac{\delta F}{\delta v}$ is called variational derivative which is defined as

$$\frac{\delta F}{\delta v} = \frac{\partial F}{\partial v} - \frac{d}{d\xi}\frac{\partial F}{\partial v'} + \frac{d^2}{d\xi^2}\frac{\partial F}{\partial v''} + \cdots.$$

By dividing each terms in Eq. (7) by f, we obtain

$$\frac{f'}{f}v' + \frac{1}{2}\frac{f'}{f}(v')^2 + v'v'' + \frac{f + f''}{f}v'' + 2\frac{f'}{f}v^{(3)} + v^{(4)} + \frac{\delta F}{f\delta v} = 0. \qquad (8)$$

Comparing Eq. (8) with Eq. (3), we have the following results

$$\frac{f'}{f} = 1, \qquad \frac{f + f''}{f} = 2, \qquad \frac{\delta F}{\delta v} = 0, \tag{9}$$

from which we obtain solution for f and F as follows:

$$f = e^{\xi}, \qquad F = 0. \tag{10}$$

Finally, we obtain the variational principle for Eq. (3) which is given by

$$L(v) = \int e^{\xi}\left[-\frac{1}{2}(v')^2 - \frac{1}{6}(v')^3 + \frac{1}{2}(v'')^2\right] d\xi. \tag{11}$$

III. CONCLUSION

In this paper, the semi-inverse method has been applied to establish the variational principle for traveling waves in a modified Kuramoto-Sivashinsky equation. Our calculations and the corresponding results confirm the simplicity and conciseness of the method.

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