Analysis of Temperature Drop Along the Radial Axis in Steady State Heat Transfer of Nuclear Fuel Element Using ANSYS APDL

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Abstract – Nuclear reactor core is usually made up of cylindrical fuel elements that contain fuel pellets, helium gas gap and cladding material. In this paper our goal was to analyze the temperature drop from the centerline of the fuel where the maximum temperature occurs, to the gas gap and then finally to the cladding surface. The geometrical, physical and thermal properties of the fuel element are known. The Nuclear fuel element used in this calculation was UO₂, which is the standard fuel element for Light Water Reactors. We used two techniques to solve this problem. The First Technique was to try to obtain an exact solution of the steady state analysis using the classical heat conduction equation. The second approach was to use FEA which is inbuilt in ANSYS APDL, where the partial differential equation is automatically solved numerically using finite different method. The results were compared and the conclusion was that the addition of gas gap between the pellet and the cladding material further contributes to the increase in the centerline temperature. The bigger the thickness of the gap, the higher the centerline temperature, this is due to the fact that the thermal conductivity of helium gas is very low, hence it is not very good at carrying heat from one surface to another. This is partly the reason why the thickness of the gas gap is usually very small.

Keywords – Nuclear Fuel Element; ANSYS APDL; FEA.

I. INTRODUCTION

The main problem of fuel rod behavior modeling is the determination of the heat transfer coefficients at the gas gap between fuel and cladding and from cladding surface to the coolant [1]. When designing a nuclear power plant, the calculation of the power produced in the reactor core and its removal by the coolant are very important. Coolant is circulated through the core and heat flows from the fuel rods to the coolant [2]. The study of heat transfer of fuel element along the radial axis is paramount to the safety of a nuclear reactor. This is due to the fact that when heat is not properly transferred between the layers of materials that make up the fuel element, the integrity of the fuel assembly structure would be compromised. This is the reason why the thermal conductivities, specific heat capacities and densities of these layers of materials that make up the fuel element are highly considered when choosing them. Though Uranium dioxide fuel is the most widely used fuel in modern nuclear power plants, it has thermal conductivity problem. The low thermal conductivity usually aid the formation of cracks in the pellet, however this negativity is equally balanced by its high melting point.

A. THERMAL CONDUCTIVITY:

Thermal conductivity of UO₂ is an essential parameter in the determination of reactor behaviour, poor thermal conductivity will create a large difference between the centerline temperature and the surface temperature during the removal of fission heat in the reactor [1]. Ironically, both UO₂ and helium gas has low thermal conductivities, that is why the cladding material which shares a contact surface with the coolant, usually has high thermal conductivity. The high thermal conductivity enables faster heat removal from the fuel element to the coolant, so that proper cooling could take place. If the cladding material has low thermal conductivity, it would create a situation where the centerline temperature gets very close to the fuel melting point, thereby reducing the available power. Thermal conductivity is the main thermal parameter when considering steady state heat conduction of fuel.

B. SPECIFIC HEAT AND DENSITY:

Apart from thermal conductivity, two other thermal parameters that play prominent role in describing heat transfer in fuel rod is...
the density and specific heat, they are more active in transient case, but less active in steady state.

**II. FORMULATION OF ANALYTICAL RESULT**

The classical equation of heat conduction in cylindrical coordinate without the axial and azimuthal terms

\[
\frac{1}{R_f} \frac{\partial}{\partial R_f} \left( R_f k_f \frac{\partial T}{\partial R_f} \right) + q_f = \rho c_p \frac{\partial T}{\partial t}
\]

(1)

Where \( \rho \) is the density, \( c_p \) is the heat capacity at constant pressure, \( k \) is the thermal conductivity and \( Q \) is the volumetric heat density in the fuel pellet.

Equation (1) is the transient equation of the fuel rod conduction.

If the conduction equation is time independent, then we have heat equation that is in steady state with internal heating \( (q'') \), hence we the poisson equation of heat conduction for the pellet and Laplace equation of heat for the cladding material.

\[
\frac{1}{R_p} d \left( R_p k_p \frac{dT_p}{dR_p} \right) + q'' = 0
\]

(2)

\[
\frac{d}{dR_p} \left( R_p \frac{dT_p}{dR_p} \right) = 0
\]

(3)

\[
\frac{d}{dR_p} \left( R_p \frac{dT_{cl}}{dR_p} \right) = 0
\]

(4)

Where \( k_p, T_p, \) and \( T_{cl} \) are the heat conductivity and temperature of the fuel pellet and temperature of the cladding.

By taking boundary conditions, we can solve the steady state case, analytically.

\[
\left( \frac{dT_p}{dR_p} \right)_{R_p=0} = 0
\]

(5)

\[
k_p \left( \frac{dT_p}{dR_p} \right)_{R_p=R_p} = k_{gg} \left( \frac{dT_{gg}}{dR_{gg}} \right)_{R_p=R_p}
\]

(6)
Analysis of Temperature Drop Along the Radial Axis in Steady State Heat Transfer of Nuclear Fuel Element Using ANSYS APDL

\[
k_{gs}\left(\frac{dT_{gs}}{dR_{fe}}\right)_{R_{gs}=R_{gs}} = k_{cl}\left(\frac{dT_{cl}}{dR_{fe}}\right)_{R_{gs}=R_{gs}}
\]

(7)

\[
T_{fp}(R_{fp}) = T_{gs}(R_{fp})
\]

(8)

\[
k_{cl}\left(\frac{dT_{cl}}{dR_{fe}}\right)_{R_{gs}=R_{gs}} = h(T_{cl}(R_{cl}) - T_{cool})
\]

(9)

\[
T_{gs}(R_{gs}) = T_{cl}(R_{gs})
\]

(10)

The boundary conditions (5), (6), (7), (8), (9) and (10) shows that (a) temperature is constant at the innermost part of the fuel pellet, hence temperature gradient is zero, (b) at the layer between the pellet outer diameter and the gas gap inner diameter, the heat flux is constant or the linear heat density is constant, (c) at the layer between the outer diameter of the gas gap and inner diameter of the cladding, the heat flux is also assumed to be constant, (d) at the boundary between pellet and the gas gap, the temperature is the same. (e) at the outer boundary between the cladding and the coolant, the thermal flux depends on the temperature difference of the cladding and the coolant, and the heat transfer coefficient of the coolant, (f) at the boundary between the gas gap and the cladding, the temperature is the same.

Solving equation (2)

\[
\frac{1}{R_{fe}}\frac{d}{dR_{fe}}\left(k_{fp}R_{fe}dT_{fp}\right) = -q^-
\]

(11)

\[
\frac{dT_{fp}}{dR_{fe}} = -\frac{q^+R_{fe}}{2k_{fp}} + \frac{\alpha}{k_{fp}R_{fe}}
\]

(12)

Applying boundary condition of equation (5), we have:

\[
\alpha = \frac{q^+R_{fe}^2}{2} = 0
\]

We therefore have:

\[
\frac{dT_{fp}}{dR_{fe}} = -\frac{q^+R_{fe}}{2k_{fp}}
\]

(10)

Solving (3)

\[
\frac{d}{dR_{fe}}\left(R_{fe}\frac{dT_{gs}}{dR_{fe}}\right) = 0
\]

\[
\frac{dT_{gs}}{dR_{fe}} = \frac{\beta}{R_{fe}}
\]

(11)

Applying boundary condition in (6) we have:

\[
k_{fp}\left(-\frac{q^+R_{fe}}{2k_{fp}}\right) = k_{gs}\left(\frac{\beta}{R_{fp}}\right)
\]

(12)
\[
\beta = -\frac{q^* R_{fp}^2}{2k_{gs}}
\]  
(13)

\[
\frac{dT_{gs}}{dR_{fe}} = -\frac{1}{R_{fe}} \frac{q^*}{2k_{gs}} R_{fp}^2
\]  
(14)

\[
T_{gs}(R_{fe}) = -\frac{q^*}{2k_{gs}} R_{fp}^2 \ln(R_{fe}) + \gamma
\]  
(15)

Solving equation (4)

\[
\frac{d}{dR_{fe}} \left( R_{fe} \frac{dT_{el}}{dR_{fe}} \right) = 0
\]  
(16)

\[
\frac{dT_{el}}{dR_{fe}} = \frac{\delta}{R_{fe}}
\]  
(17)

Applying boundary condition in (7) we have:

\[
k_{gs} \left( \frac{dT_{gs}}{dR_{fe}} \right)_{R_{fe}=R_{gs}} = k_{cl} \left( \frac{dT_{el}}{dR_{fe}} \right)_{R_{fe}=R_{gs}}
\]  
(18)

\[
k_{gs} \left( -\frac{1}{R_{gs}} \frac{q^*}{2k_{gs}} R_{fp}^2 \right) = k_{cl} \left( \frac{\delta}{R_{gs}} \right)
\]  
(19)

\[
\delta = -\frac{q^* R_{fp}^2}{2k_{cl}}
\]  
(20)

\[
\frac{dT_{el}}{dR_{fe}} = -\frac{q^* R_{fp}^2}{2k_{cl} R_{fe}}
\]  
(21)

\[
T_{el}(R_{fe}) = -\frac{q^*}{2k_{cl}} R_{fp}^2 \ln(R_{fe}) + \varepsilon
\]  
(22)

Integrating equation (10)

\[
T_{fp}(R_{fe}) = -\frac{q^*}{4k_{fp}} R_{fp}^2 + \eta
\]  
(23)

Applying the boundary condition in equation (8)

\[
-\frac{q^*}{4k_{fp}} R_{fp}^2 + \eta = -\frac{q^*}{2k_{gs}} R_{fp}^2 \ln(R_{fp}) + \gamma
\]  
(24)
Applying the boundary condition in equation (9)

\[ k_{cl} \left( \frac{q''}{2k_{cl}R_{fp}} R^2_{fp} \right) = h \left( -\frac{q''}{2k_{cl}} R^2_{fp} \ln(R_{fp}) + \varepsilon - T_{cool} \right) \]

\[ \varepsilon = \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln(R_{cl}) + T_{cool} \]

Hence the cladding temperature distribution is given thus:

\[ T_{cl}(R_{fe}) = \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln\left( \frac{R_{cl}}{R_{fe}} \right) + T_{cool} \]

Applying the boundary condition in equation (10)

\[ -\frac{q''}{2k_{gg}} R^2_{fp} \ln\left( \frac{R_{gg}}{R_{fp}} \right) + \gamma = \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln\left( \frac{R_{cl}}{R_{gg}} \right) + T_{cool} \]

\[ \gamma = \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln\left( \frac{R_{cl}}{R_{gg}} \right) + \frac{q''}{2k_{gg}} R^2_{fp} \ln\left( \frac{R_{gg}}{R_{fp}} \right) + T_{cool} \]

The gas gap temperature distribution is given thus:

\[ T_{gg}(R_{fe}) = \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln\left( \frac{R_{cl}}{R_{gg}} \right) + \frac{q''}{2k_{gg}} R^2_{fp} \ln\left( \frac{R_{gg}}{R_{fp}} \right) + T_{cool} \]

From (23), we have

\[ -\frac{q''}{4k_{fp}} R^2_{fp} + \eta = \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln\left( \frac{R_{cl}}{R_{gg}} \right) + \frac{q''}{2k_{gg}} R^2_{fp} \ln\left( \frac{R_{gg}}{R_{fp}} \right) + T_{cool} \]

\[ \eta = \frac{q''}{4k_{fp}} R^2_{fp} + \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln\left( \frac{R_{cl}}{R_{gg}} \right) + \frac{q''}{2k_{gg}} R^2_{fp} \ln\left( \frac{R_{gg}}{R_{fp}} \right) + T_{cool} \]

From (22), we have

The fuel pellet temperature distribution is given thus:

\[ T_{fp}(R_{fe}) = -\frac{q''}{4k_{fp}} R^2_{fp} + \frac{q''}{4k_{fp}} R^2_{fp} + \frac{q''}{2hR_{cl}} R^2_{fp} + \frac{q''}{2k_{cl}} R^2_{fp} \ln\left( \frac{R_{cl}}{R_{gg}} \right) + \frac{q''}{2k_{gg}} R^2_{fp} \ln\left( \frac{R_{gg}}{R_{fp}} \right) + T_{cool} \]

Fuel geometrical and thermal parameters used for this validation exercise are as follows:
### Table 1: Table Of Parameters

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volumetric heat density of fuel ($q''$)</td>
<td>4000000 W/m³</td>
</tr>
<tr>
<td>Heat transfer coefficient ($h$)</td>
<td>100 W/m²K</td>
</tr>
<tr>
<td>Fuel pellet radius ($R_{fp}$)</td>
<td>0.015 m</td>
</tr>
<tr>
<td>Cladding radius ($R_{cl}$)</td>
<td>0.0181 m</td>
</tr>
<tr>
<td>Gas gap radius ($R_{gg}$)</td>
<td>0.0151 m</td>
</tr>
<tr>
<td>Gas gap thickness ($R_{gg} - R_{fp}$)</td>
<td>0.0001 m</td>
</tr>
<tr>
<td>Cladding thickness ($R_{cl} - R_{gg}$)</td>
<td>0.003 m</td>
</tr>
<tr>
<td>Fuel element length ($H$)</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Coolant temperature ($T_{cool}$)</td>
<td>300 °C</td>
</tr>
<tr>
<td>Density of the fuel ($\rho_1$)</td>
<td>10950 kg/m³</td>
</tr>
<tr>
<td>Density of cladding ($\rho_{cl}$)</td>
<td>6550 kg/m³</td>
</tr>
<tr>
<td>Specific heat capacity of fuel ($c_{p1}$)</td>
<td>236 J/kgK</td>
</tr>
<tr>
<td>Specific heat capacity of cladding ($c_{pcl}$)</td>
<td>285.8 J/kgK</td>
</tr>
<tr>
<td>Thermal conductivity of fuel ($k_{fp}$)</td>
<td>6 W/mK</td>
</tr>
<tr>
<td>Thermal conductivity of gas gap ($k_{gg}$)</td>
<td>0.1513 W/mK</td>
</tr>
<tr>
<td>Thermal conductivity of cladding ($k_{cl}$)</td>
<td>21.5 W/mK</td>
</tr>
</tbody>
</table>

### Table 2: Definition Of Symbol Of Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{fp}$ ($R_{fp}$)</td>
<td>Temperature distribution of the fuel pellet</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{gg}$ ($R_{gg}$)</td>
<td>Temperature distribution of the gas gap</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{cl}$ ($R_{cl}$)</td>
<td>Temperature distribution of the cladding</td>
<td>°C</td>
</tr>
<tr>
<td>$k_{fp}$</td>
<td>Thermal conductivity of fuel pellet</td>
<td>W/mK</td>
</tr>
<tr>
<td>$k_{gg}$</td>
<td>Thermal conductivity of the gas gap</td>
<td>W/mK</td>
</tr>
<tr>
<td>$k_{cl}$</td>
<td>Thermal conductivity of cladding</td>
<td>W/mK</td>
</tr>
<tr>
<td>$\alpha, \beta, \gamma, \delta, \epsilon, \eta$</td>
<td>Integration constants</td>
<td></td>
</tr>
<tr>
<td>$R_{fp}$</td>
<td>Radius of the fuel pellet</td>
<td>m</td>
</tr>
<tr>
<td>$R_{fe}$</td>
<td>Radius of the fuel element</td>
<td>m</td>
</tr>
<tr>
<td>$R_{gg}$</td>
<td>Radius of the gas gap</td>
<td>m</td>
</tr>
<tr>
<td>$R_{cl}$</td>
<td>Radius of the cladding</td>
<td>m</td>
</tr>
<tr>
<td>$h$</td>
<td>Coefficient of heat transfer</td>
<td>W/m²K</td>
</tr>
<tr>
<td>$q''$</td>
<td>Volumetric heat generation rate</td>
<td>W/m³</td>
</tr>
<tr>
<td>$T_{cool}$</td>
<td>Coolant temperature</td>
<td>°C</td>
</tr>
</tbody>
</table>

### III. Analytical Results

We can now plot the temperature as a function of radius to observe how it changes within the fuel element from the pellet to the gas gap and then to the cladding, especially at the point where the pellet and the gas gap overlap and where the gas gap and cladding overlap. In this work, our fuel element is assumed to have infinite length. This is the essence of the boundary condition imposed to ease the analytical calculation. The resulting graph
below showed a good behavior of the model, which will be validated using ANSYS APDL.

The PTC-MATHCAD toolbox was used to compute and plot the analytical solution; it is a user-friendly computing environment with a lot of symbolic solution which provides an accurate analysis of result. As can be seen from the graph below, the plotting is quite simple with simple labeling system. Therefore, I can say that while PTC-MATHCAD helped to solve the analytical solution, ANSYS APDL assisted with the numerical simulation result.

Using PTC-MATHCAD worksheet, we plotted the analytical results thus:

![Figure 2. Temperature distribution of fuel element](image)

**IV. NUMERICAL SIMULATION OF THE STEADY STATE CONDUCTION OF NUCLEAR FUEL ROD USING ANSYS APDL**

The parameter of the fuel rod are as stated earlier, the simulation was done on 3-D platform.

![Figure 3. Temperature distribution contour](image)  
![Figure 4. Temperature distribution graph](image)
V. DISCUSSION OF RESULTS

From the results obtained above, we can say that we have been able to use method of computation to investigate the temperature in steady state heat conduction of nuclear fuel element. In the analytical calculation, we formed the model based on certain boundary condition that was derived previously, using these boundary conditions. We were able to obtain the graphical representation using PTC-MATHCAD Tool. In the second calculation, we used ANSYS APDL platform where we selected thermal analysis, 8 node 78 as element type, inputted the thermal conductivities for the three layers of material, rectangle as element geometry, meshing tool, defined loads and the solved the current LS. The solution obtained was in good agreement with the analytical solution.

VI. CONCLUSION

In this paper, one of the major observations was the increase in the center temperature with increase thickness of the gas gap. This observation was noticed in the Simulation using ANSYS APDL. The increase in the centerline temperature observed when the gas gap thickness is increased, was due to the low thermal conductivity of helium used for the gap. Hence in the fabrication of conventional nuclear fuel element, the gap is usually very tiny to minimize the rate of increase of the centerline temperature while also serving its purpose. Hence we observed that if the fuel element were modeled using the same parameters as this but without the tiny gas gap, the maximum temperature would decrease. The rate of increase or decrease will depend on the rate of increase or decrease of the gas gap thickness.

The two results obtained from the two methods of calculation validate each other.

REFERENCES

[3] Philipp Hangi, "Investigating BWR stability with a new linear frequency domain method and detailed 3-D neutronics".