

Quinquinomial Power Functions With One Phase Of The Dwell In Cam Mechanisms

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Abstract – A family of quinquinomial power motion laws without finite and infinite spikes with better characteristics has been compiled and studied than in the case of analogous quinquinomial motion laws. They are suitable for cases where polydyne cam mechanisms are synthesised with only one phase of dwell at the follower. The aim is to compile and investigate a family without a finite and infinite spikes quinquinomial power laws of motion with better characteristics than similar quinquinomial power laws of motion, which are suitable for the synthesis of cam mechanisms with phases of dwell. The composite family without a finite and infinite spikes quinquinomial power laws of motion are suitable for cases where polydyne cam mechanisms are synthesized with only one phase of dwell at the follower. Extreme values of output velocities and accelerations are reduced, thus reducing the inertial loading of the mechanisms. The derived family of quinquinomial power normalized functions for modeling the laws of motion without a finite and infinite spikes of the cam mechanisms enables us to achieve better dynamic characteristics in comparison with other laws of motion for polydyne cams, in cases where one of the phases of dwell of the follower is missing.

Keywords – Cam Mechanisms, Laws Of Motion, Power Functions

I. INTRODUCTION

The laws of motion of polydyne cam mechanisms are smooth, non-peaked functions at least up to the fourth derivative of the output motion, to avoid acceleration peaks in high-speed and elastic mechanical systems [1], [2], [3], [4], such as are the cam-lever distribution valves mechanisms of automotive engines [5], [6], [7] and many other modern mechanical systems of various technological machines [8], [9], [10], [11], [12]. This advantage of the laws of motion is due to the greater extreme values of the output velocities and accelerations. These values can be significantly reduced in single-phase cam mechanisms [13] if the output acceleration has the same ultimate value in the transition from the return phase (reverse move, return stroke) to the distance phase (rise phase, outstroke phase) of the follower (in the lack of a high dwell) or in the transition from rise phase to the return phase of the follower (in the lack of a low dwell).

Otherwise, at zero acceleration in the transition from one phase to another of motion of follower, the acceleration rapidly changes from one extreme value to zero and again to an extreme value in the next phase of motion, with significant elastic vibrations possible.

The aim is to compile and investigate a family without finite and infinite spikes quinquinomial power laws of motion with better characteristics than similar quinquinomial power laws of motion, which are suitable for the synthesis of cam mechanisms with phases of dwell. The composited family without a finite and infinite spikes quinquinomial power laws of motion is suitable for cases where polydyne cam mechanisms are synthesized with only one phase of dwell at the follower [14].

II. TWO CASES FOR GEOMETRIC CYCLE WITH ONE PHASE OF DWELL

A condition for deriving power laws of motion of polydyne cam mechanisms with one phase of dwell is the use of basic normalized functions with at least five terms, the first of which must be of exponent 2 ($k = 2$) and the others exponents of five upwards.

The normalized function and its derivatives for the rise and return phases $\Phi_1 = \Phi_3 = \Phi$ for an argument $\xi = \varphi / \Phi \in [0; 1]$ can be described by an output normalized function and its derivatives $u'(\xi), u''(\xi), u'''(\xi)$ and $u''''(\xi)$. There are two possible cases:

Case 1. One cycle involves only a phase of high dwell Φ_2 (Figure 1a);

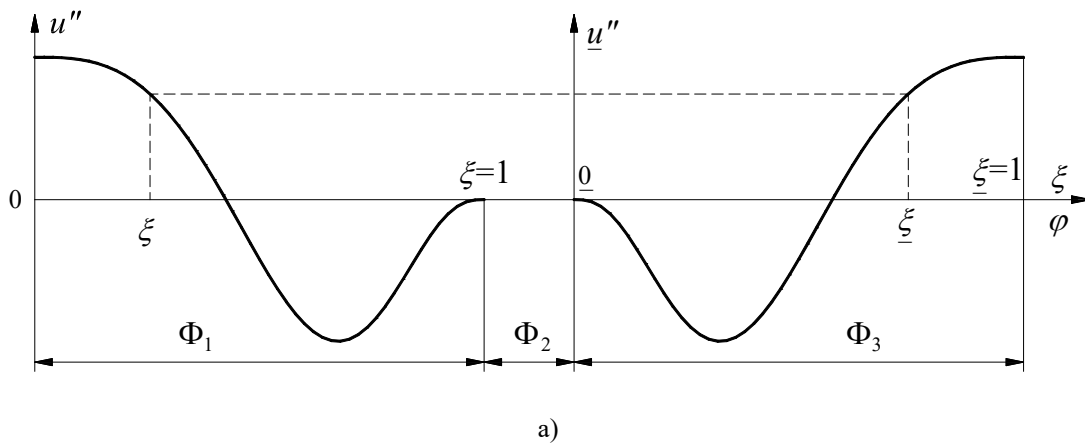
Case 2. One cycle involves only a phase of low dwell Φ_4 (Figure 1b).

In both cases, an argument $\underline{\xi} = 1 - \xi$, is introduced, which follows:

$$\begin{cases} \underline{u}(\underline{\xi}) = u(1 - \xi); \underline{u}'(\underline{\xi}) = -u'(1 - \xi); \underline{u}''(\underline{\xi}) = u''(1 - \xi); \\ \underline{u}'''(\underline{\xi}) = -u'''(1 - \xi); \underline{u}''''(\underline{\xi}) = u''''(1 - \xi) . \end{cases} \tag{1}$$

In case 1 ($\Phi_4 = 0$), for the distance phase Φ_1 , the normalized function and its derivatives coincide with the output normalized function $u(\xi)$ and its derivatives $u'(\xi), u''(\xi), u'''(\xi)$ and $u''''(\xi)$. In the return phase Φ_3 the argument is $\underline{\xi} = 1 - \xi$ and equations (1) are used.

In case 2 ($\Phi_2 = 0$) for the return phase Φ_3 , the normalized function and its derivatives coincide with the output normalized function $u(\xi)$ and its derivatives $u'(\xi), u''(\xi), u'''(\xi)$ and $u''''(\xi)$. In the distance phase Φ_1 the argument is $\underline{\xi} = 1 - \xi$ and equations (1) are used.



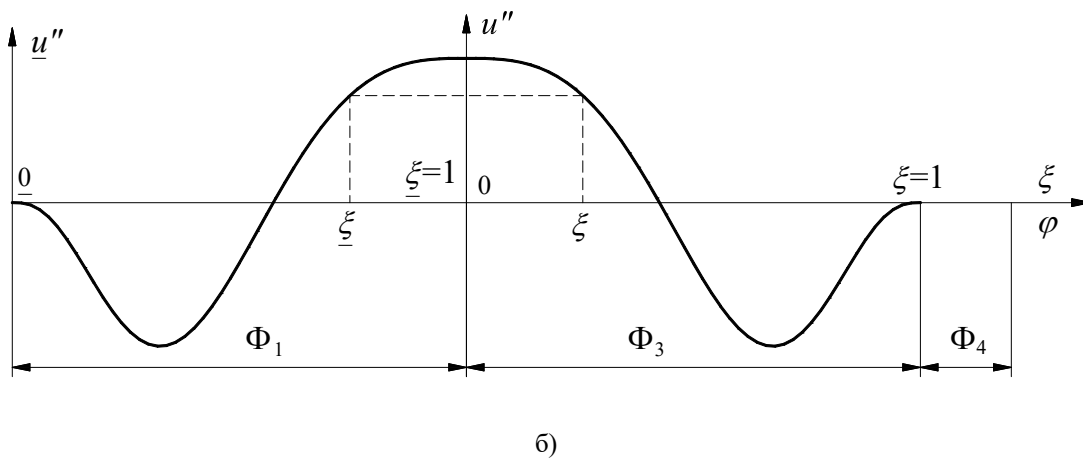


Fig. 1. Coupling the function with for one geometric cycle in one phase of dwell:

a) high; b) low

Under these conditions, the analogs of the accelerations $u''(\xi)$ and $\underline{u}''(\underline{\xi})$ are coupling without jump in the transition between the phases of the rise and the return of the follower (Figure 1).

III. DERIVATION OF POWER POLYNOMIALS

By the well-known formula [15] and selected values of $j = k, m, p, q, s$ of a polynomial of type $u(\xi) = \sum_j a_j \xi^j$, the coefficients a_k, a_m, a_p, a_q, a_s of the polynomial can be determined.

Let the values of the exponents k, m, p, q, s be chosen by the formula $z = 2 + in$, where z is a generalized registration of a series of exponents m, p, q, s , respectively at $i = 1, 2, 3, 4$ and $n = 3, 6, 9, 12$. A family of four polynomials and the corresponding power exponents are presented in Table 1, with $k = 2$ for each polynomial. Thus, for the coefficients a_k, a_m, a_p, a_q, a_s , the values recorded in Table 2 are calculated.

Table 1. Exponents k, m, p, q, s

polynomial i	n	m	p	q	s
1	3	5	8	11	14
2	6	8	14	20	26
3	9	11	20	29	38
4	12	14	26	38	50

Table 2. Coefficients a_k, a_m, a_p, a_q, a_s

polynomial	a_k	a_m	a_p	a_q	a_s
1	$\frac{770}{243}$	$-\frac{1232}{243}$	$\frac{1155}{243}$	$-\frac{560}{243}$	$\frac{110}{243}$

2	$\frac{455}{243}$	$-\frac{455}{243}$	$\frac{390}{243}$	$-\frac{182}{243}$	$\frac{35}{243}$
3	$\frac{30305}{19683}$	$-\frac{22040}{19683}$	$\frac{18183}{19683}$	$-\frac{8360}{19683}$	$\frac{1595}{19683}$
4	$\frac{43225}{31104}$	$-\frac{24700}{31104}$	$\frac{19950}{31104}$	$-\frac{9100}{31104}$	$\frac{1729}{31104}$

• For power polynomial №1 with coefficients of row №1 of table 2 and its derivatives, are obtained:

$$\left\{ \begin{aligned} u &= \frac{1}{243} (770\xi^2 - 1232\xi^5 + 1155\xi^8 - 560\xi^{11} + 110\xi^{14}), \\ u' &= \frac{1540}{243} (\xi - 4\xi^4 + 6\xi^7 - 4\xi^{10} + \xi^{13}), \\ u'' &= \frac{1540}{243} (1 - 16\xi^3 + 42\xi^6 - 40\xi^9 + 13\xi^{12}), \\ u''' &= \frac{6160}{81} (-4\xi^2 + 21\xi^5 - 30\xi^8 + 13\xi^{11}), \\ u^{(4)} &= \frac{6160}{81} (-8\xi + 105\xi^4 - 240\xi^7 + 143\xi^{10}). \end{aligned} \right. \quad (2)$$

• For power polynomial №2 with coefficients of row №2 of table 2 and its derivatives are obtained:

$$\left\{ \begin{aligned} u &= \frac{1}{243} (455\xi^2 - 455\xi^8 + 390\xi^{14} - 182\xi^{20} + 35\xi^{26}), \\ u' &= \frac{910}{243} (\xi - 4\xi^7 + 6\xi^{13} - 4\xi^{19} + \xi^{25}), \\ u'' &= \frac{910}{243} (1 - 28\xi^6 + 78\xi^{12} - 76\xi^{18} + 25\xi^{24}), \\ u''' &= \frac{7280}{81} (-7\xi^5 + 39\xi^{11} - 57\xi^{17} + 25\xi^{23}), \\ u^{(4)} &= \frac{7280}{81} (-35\xi^4 + 429\xi^{10} - 969\xi^{16} + 575\xi^{22}). \end{aligned} \right. \quad (3)$$

• For power polynomial №3 with coefficients of row №3 of table 2 and its derivatives, are calculated:

$$\left\{ \begin{aligned} u &= \frac{1}{19683}(30305\xi^2 - 22040\xi^{11} + 18183\xi^{20} - 8360\xi^{29} + 1595\xi^{38}), \\ u' &= \frac{60610}{19683}(\xi - 4\xi^{10} + 6\xi^{19} - 4\xi^{28} + \xi^{37}), \\ u'' &= \frac{60610}{19683}(1 - 40\xi^9 + 114\xi^{18} - 112\xi^{27} + 37\xi^{36}), \\ u''' &= \frac{60610}{2187}(-40\xi^8 + 228\xi^{17} - 336\xi^{26} + 148\xi^{35}), \\ u'''' &= \frac{242440}{2187}(-80\xi^7 + 969\xi^{16} - 2184\xi^{25} + 1295\xi^{34}). \end{aligned} \right. \tag{4}$$

• For power polynomial №4 with coefficients of row №4 of table 2 and its derivatives, are obtained:

$$\left\{ \begin{aligned} u &= \frac{1}{31104}(43225\xi^2 - 24700\xi^{14} + 19950\xi^{26} - 9100\xi^{38} + 1729\xi^{50}), \\ u' &= \frac{86450}{31104}(\xi - 4\xi^{13} + 6\xi^{25} - 4\xi^{37} + \xi^{49}), \\ u'' &= \frac{86450}{31104}(1 - 52\xi^{12} + 150\xi^{24} - 148\xi^{36} + 49\xi^{48}), \\ u''' &= \frac{129675}{3888}(-52\xi^{11} + 300\xi^{23} - 444\xi^{35} + 196\xi^{47}), \\ u'''' &= \frac{129675}{972}(-143\xi^{10} + 1725\xi^{22} - 3885\xi^{34} + 2303\xi^{46}). \end{aligned} \right. \tag{5}$$

The derivative functions u' , u'' and u''' of all polynomials are zeroed for the interval boundaries $\xi \in [0, 1]$, as can be seen from the graphs in Figure 2. These polynomials make it possible to generate uniform acceleration motion of the output unit at a considerable interval of change of the input coordinate. This interval increases with increasing polynomials exponents, as can be seen from Figure 2.

In table 3 of the extremums of functions u' , u'' and u''' , comparison is made.

Table 3. Comparison table of the extremums of functions u' , u'' and u'''

normalized polynomial $u(\xi)$	u'_{\max}	u''_{\max}	u''_{\min}	u'''_{\max}	u'''_{\min}
polynomial 1 (2)	1.957	6.337	-6.309	33.03	-35.859
polynomial 2 (3)	1.86	3.745	-8.386	75.515	-48.149
polynomial 3 (4)	1.848	3.079	-10.769	138.079	-73.247
polynomial 4 (5)	1.85	2.779	-13.226	220.249	-106.376

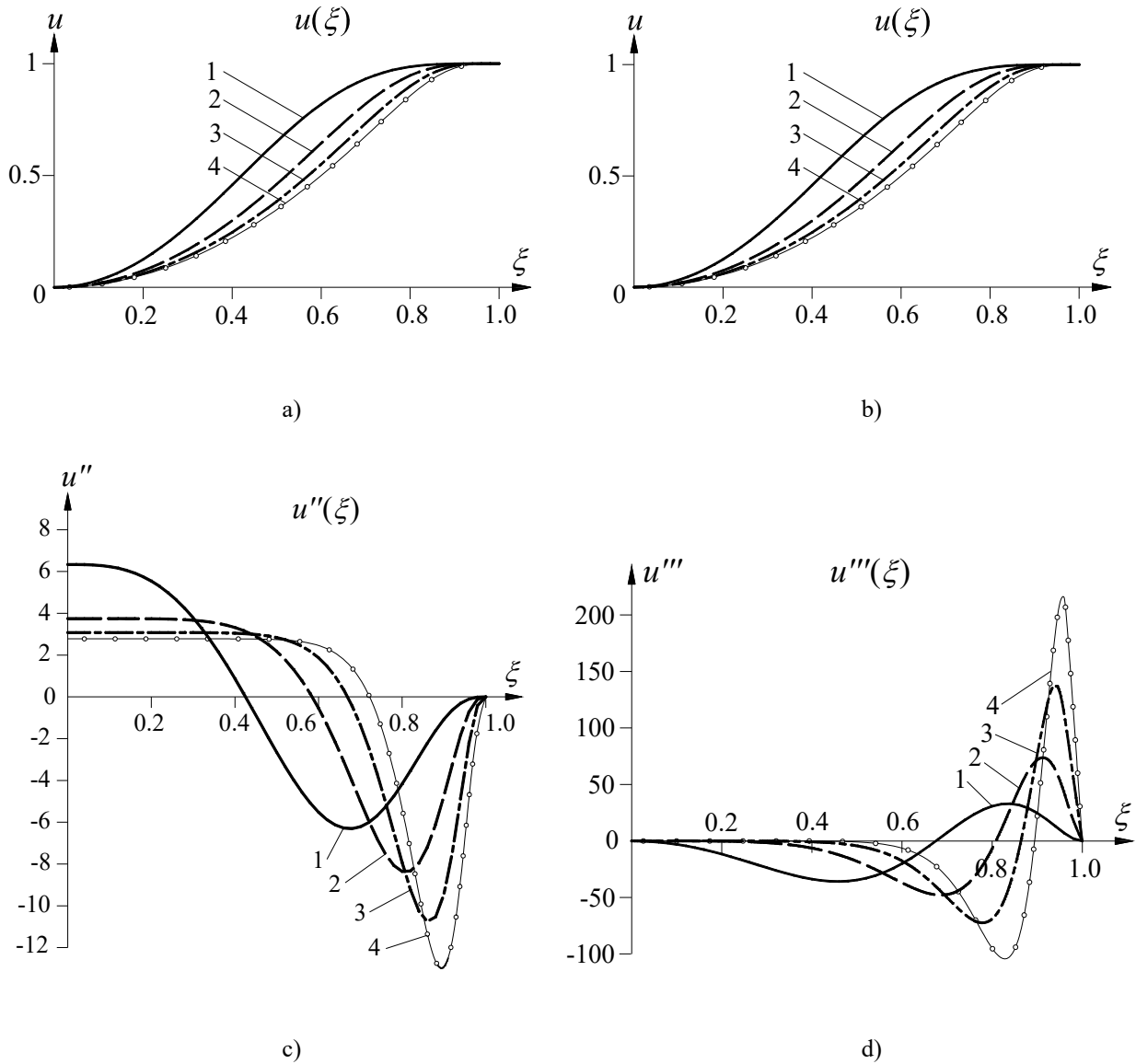


Fig. 2. Graphs of polynomials 1, 2, 3 и 4: a) $u(\xi)$; b) $u'(\xi)$; c) $u''(\xi)$; d) $u'''(\xi)$

A detailed solution to the question of the laws of motion and synthesis of cam mechanisms was made by Galabov, Roussev, and Paleva-Kadiyska in [16].

IV. CONCLUSION

The derived family of quinquinomial power normalized functions for modeling the laws of motion without a finite and infinite spikes of the cam mechanisms enables us to achieve better dynamic characteristics in comparison with other laws of motion for polydyne cams, in cases where one of the phases of dwell of the follower is missing.

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