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Analysis of Massive Multiple Antenna System Capacity based on Channel State Information Performance

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Abstract—This paper has simulated channel capacity with respect to information transmitted without and with channel state information (CSI) in multiuser massive multiple input multiple output (MU massive MIMO) for downlink wireless communication. The objective was to examine the effect of CSI at the transmitter on the channel capacity. Simulations were conducted considering different base station antenna configurations transmitting signals to multiple users' terminals over Rayleigh channels at signal to noise ratio (SNR) of 0 to 25. Two scenarios were considered namely, channel capacity of uninformed transmitter (that is without channel state in information known at the transmitter) and channel capacity of informed transmitter (with channel state information known at the transmitter). Power allocation among the transmit antennas was done using the water filling algorithm. The results of the simulations conducted revealed that the capacity of the system was improved when the transmitter was aware of the CSI. Also, increased number of receive antennas (UEs) was observed to compensate for lack in channel knowledge. Furthermore, increasing the number of base station antennas resulted in more channel capacity.

Keywords— Base station, channel capacity, channel state information, MU massive MIMO, Water filling algorithm

I. Introduction

The need to offer high data rate for each user is the objective of wireless communication improvement. Presently, Multi-User multiple input multiple output (MU-MIMO) system is the technique used in wireless communication [1]. The performance improvement of the system in terms of data throughput and link can be theoretically achieved by increasing the number of transmit or receive antennas. In addition, MU-MIMO system does not only provide improvement in data throughput and link reliability but also facilitates in the saving of transmitter energy due to the array gain [2]. The fact that Multi-User (MU) systems provide the opportunity to transmit signal at the same time to numerous users makes the benefits more attractive [3]. The base station (BS) is equipped with multiple antennas and serves many users in a MU-MIMO system. Communication between the based station and several users is usually through orthogonal channels. This means that the base station communicates with every user in a separate time and frequency resource [2]. Nevertheless, if the base station communicates with every user in the same time-frequency signals, a higher data rate can be achieved.

Massive MIMO is a multiple antenna technique that aims to improve data transmission rate using hundreds of transmit antennas. This means that the more the BS or transmit antennas employed, the more the data streams that can be transmitted to serve more user equipments (UEs) or terminals and thereby minimizing the radiated power, even as the data rate is boosted. Also, the reliability of the link is improved through spatial diversity and, offers additional degree of freedom (DOF), and enhances the performance regardless of the measurement noisiness. As a result of the wide range of states of freedom and greater selectivity in data stream transmission and reception possessed by massive MIMO, cancellation of interference is enhanced [4]. In massive MIMO, transmission into undesired directions to reduce negative interference that causes low latency can be fairly averted by BS. Furthermore, proper use of beamforming schemes by massive MIMO minimizes fading drops, which additionally improves signal to noise ratio (SNR), bit rate and reduces latency [5].

The propagation of signal from transmitter to receiver experiences multipath effect such as shadowing, scattering, fading, and path loss. Hence, it is important to know the channel state information (CSI) in both forward and reverse links so as to achieve successful transmission at different conditions [4]. This study examines the channel capacity of MU massive MIMO system for downlink operation considering two conditions: transmission without and with CSI.

II. System Channel

This section considers the mathematical definition of downlink data transmission. Now, consider the simplified down link massive multiple antenna system in a single cell shown in Fig. 1.

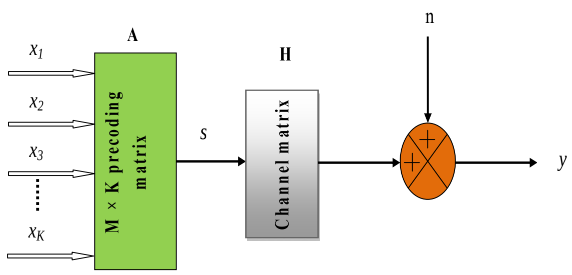

Fig. 1 A model of downlink massive multiple antenna system in a single cell

Assuming a perfect channel given by $K \times M$ channel matrix $H = \left[h_1^T \quad \cdots \quad h_K^T\right]^T$, where h_K respresents the $1 \times M$ channel vector among K^{th} user equipment (UE) and base station. With the propagation channel usually designed by assuming large-scale and small-scale fading [6], small-scale fading has been considered in this work assuming the elements of H defined as the channel matrix among base station antennas and users' equipment given by (1) are separately and identically distributed Gaussian distribution with unit variance and zero mean.

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \cdots & h_{1M} \\ h_{21} & h_{22} & h_{23} & \cdots & h_{2M} \\ h_{31} & h_{32} & h_{33} & \cdots & h_{3M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{K1} & h_{K2} & h_{K3} & \cdots & h_{KM} \end{bmatrix} \tag{1}$$

Since the K number of users will receive their respective information signal from the vector $K \times 1$, so the received signal is defined by the input-output relationship of the massive multiuser multiple antenna channel considering Fig. 1 and is given by:

$$y = Hs + n \tag{2}$$

where s denotes the $M \times 1$ vector of precoded transmitted symbols (or signals), y is the $K \times 1$ vector of signals received by UEs, n is $K \times 1$ user equipment additive white Gaussian noise (AGWN) vector having independently and identically distributed Gaussian distribution with zero-mean and with variance N_0.

Assuming linear precoding at the base station for both effectiveness and analytical simplicity, s in (2) can be defined looking at Fig. 1 without considering the power of the transmitted signal by:

$$s = Ax \tag{3}$$

where A represents a $M \times K$ precoding matrix, and x denotes input vector such that $x = [x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_K]^T$ with x_k the data symbol of k^{th} user which is assumed independently and identically distributed Gaussian distribution with zero-mean with unit variance.

Considering the precoding matrix having $\|A\| = 1$ [6] and P_s representing the transmitted power from the base station (that is the total average energy available at the base station in one symbol period). Hence, the transmitted signal from the base station is defined such that (3) becomes:

$$s = \sqrt{P_s} A x \tag{4}$$

Substituting Equation (4) into Equation (2) gives:

$$y = \sqrt{P_s} HAx + n \tag{5}$$

Using the notation $A = [a_1 \quad a_2 \quad a_3 \quad \cdots \quad a_k]$, the transmitted signal on the i^{th} antenna can be expressed by:

$$s_i = \sqrt{P_s} \sum_{j=1}^{k} a_{ij} x_j \tag{6}$$

The k^{th} user equipment received signal can be defined by:

$$y_k = \sqrt{P_s} \sum_{j=i}^{K} \sum_{m=1}^{M} h_{k,m} a_{m,j} x_j + n_k \tag{7}$$

Looking at (7), the desired signal part when $j = k$ and the interference part can be expressed by:

$$y_k = \underbrace{\sqrt{P_s} \sum_{m=1}^{M} h_{k,m} a_{m,k} x_k}_{\text{desired signal}} + \underbrace{\sqrt{P_s} \sum_{j \neq k}^{K} \sum_{m=1}^{M} h_{k,m} a_{m,j} x_j}_{\text{interference}} + \underbrace{n_k}_{\text{noise}} \tag{8}$$

It can be seen from (8) that received signal of each user (that is k^{th} user equipment) consists of desired signal, interference known as multiuser interference (MUI), and noise (which is an AWGN) having zero-mean with variance $N_0 = 1$, such that $n_k \in CN(0,1)$ for all users.

Therefore, the received signal-to-interference –plus-noise ratio (SINR) of the k^{th} user can be written as given by:

$$SINR_k = \frac{\sqrt{P_s} \sum_{m=1}^{M} h_{k,m} a_{m,k} x_k}{\sqrt{P_s} \sum_{j \neq k}^{K} \sum_{m=1}^{M} h_{k,m} a_{m,j} x_j + n_k} \tag{9}$$

Equation (9) is a function of transmit precoding matrix a_k and x_k, which is transmit signal vector for k^{th} user. The same holds for all K users connected in downlink multiuser multiple antenna system.

III. CHANNEL CAPACITY

In this section, the capacity of the system is examined considering the downlink technique exploiting the channel state information (CSI) on the transmitter side. Utilization of such channel information allows the channel capacity to increase, error performance improvement, and at the same time reduce complexity of hardware. The capacity C of a MIMO system can be defined theoretically by the following formula stated in Telatar [7] and cited by Saad et al. [8]:

$$C = E_H\left[\log_2 \det\left(I_{N_r} + \frac{P_s}{MN_0} HAH^*\right)\right] \tag{10}$$

where A is the convariance matrix of the input and P_s is the sum of the tramit power, N_o is noise power in each receive side antenna. Hence, the mathematical expressions for uninformed transmitter (without channel state information tranmitter) and informed transmitter (with channel state information transmitter) are presented as follows.

1.1 Without Channel State Information Transmission

At any time the state of the channel is unknown to the transmitter, though well known to the receiver, the finest option is to equally divide the available transmit power among the antennas of the transmitter. Assuming all the components of the vector x transmitted are statistically independent, which means that $A = I_M$ with Gaussian distribution, then the ergodic capacity is given by Saad et al.[8]:

$$C = E_H = \left[\log_2 \det\left(I_{N_r} + \frac{P_s}{MN_0} HH^*\right)\right] \tag{11}$$

Given the conformity to the Eigen Value Decomposition theorem, $HH^* = VDV^*$, the MIMO channel capacity can be defined as:

$$C = E_H\left[\log_2 \det\left(I_{N_r} + \frac{P_s}{MN_0} VDV^*\right)\right] \tag{12}$$

where V is $N_r \times M$ matrix that respresents the eigen vectors of the channel H satisfying $VV^* = V^*V = I_{N_r}$ and $D = \text{diag}\{\lambda_1, \lambda_2, \cdots \lambda_{N_r}\}$, $\lambda_i \geq 0$. The matrix D is a diagonal matrix that consists of eigenvalues, λ_i of the channel H. Applying the identity relationship, say for two matrices A and B, such that A is m×n matrix and B is n×m, $\det(I_m + AB) = \det(I_n + BA)$, (12) can be reduced to:

$$C = E_H\left[\log_2 \det\left(I_{N_r} + \frac{P_s}{MN_0} D\right)\right]$$

(13)

or equally as:

$$C = E_H\left[\sum_{i=1}^{r} \log_2\left(1 + \frac{P_s}{MN_0} \lambda_i\right)\right] \tag{14}$$

where, $r = \text{rank}(HH^*) = \min[M, N_r]$ (number of parallel channels) and λ_i such that $(i = 1, \cdots, R)$, are the positive eigenvalues of HH^*. The channel capacity of the MIMO system is expressed in (3) as the aggregate of the capacities of R Single Input Single Output (SISO) channels with each having power gain λ_i and transmits power P_s/M. It is observed that all eigenchannels in (14) are assigned the same power since theses channels are not accessible as a result of the fact that the transmitter has no channel state information (that is the transmitter is uninformed) hence, the transmitter divides the power equally among the channels.

1.2 With Channel State Information Transmission

It is posible for the transmitter to have the knowledge of the CSI or channel matrix H prior to transmitting the data vector. For example, in Time Division Duplexing (TDD) schemes, a feedback mechanism can be established to send CSI back to the transmitter from the receiver [8, 9] as shown in Fig. 2. In such case, the channel capacity of the MIMO system is increased by resorting to Water Filling algorithm [10] by allocating various levels of transmit power to the various M transmitting antennas. The power allocation is doned on the bases that the better the channel, the more power is geiven to it and vice versa.

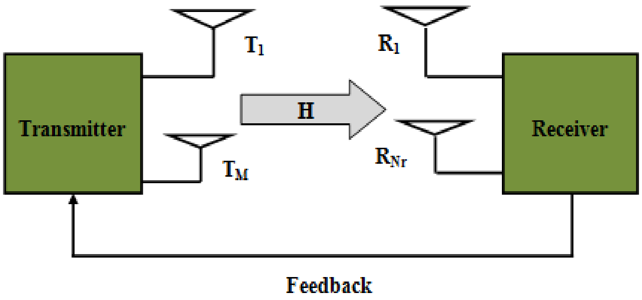

Fig.. 2 Channel state information feedback

1.3 Water Filling Algorithm

Assuming that the parameters of the channel are known at the transmitter, water filing algorithm can be utilized in minimizing the capacity of the channel by assigning more power to the channels in good state and less or nothing to the channels in bad state [10]. For an ideal channel, the system can be defined as in (2). Thus, given that the channel matrix $H = USV^*$ (that is Singular Value Decomposition, SVD, of H) the system can be expressed as:

$$y = USV^* x + n \tag{15}$$

Or writen as:

$$y = svd(H)x + n \tag{16}$$

where U is a matrix that contains the eigenvectors of the receiver, V is a matrix that contains the eigenvectors of the transmitter and S is a diagonal matrix that contains the non-zero singular values of channel matrix H. The matrices U and V are unitary and stisfy: $UU^* = U^*U = I_{Nr}$ and $VV^* = V^*V = I_M$.

The transmitted vector is multiplied by a matrix V before transmission to eliminate the effect of matrix V^* contained in channel matrix H. Similarly, the received vector is multiplied by a matrix U^* to eliminate the effect of matrix U contained in channel matrix H. If the matrix U is used to divide (15) such that $x' = V^* x, y' = U^* y, n' = U^* n$, (15) can be written as:

$$y' = Sx' + n' \tag{17}$$

Equation (17) represnts a set of parallel SISO channels whose power gains are non-zero singular digonal values of matrix S.

The MIMO channel capacity is the aggregate of the capacities of each parallel SISO channel and is expresed as:

$$C = \left[\sum_{i=1}^{r} \log_2 \left(1 + \frac{P_s \vartheta_i}{MN_0} \lambda_i \right) \right] \tag{18}$$

where ϑ_i is the amount of power transmitted over eigenvalue λ_i such that [8]:

$$\sum_{i=1}^{r} \vartheta_i = M \tag{19}$$

The implication of the channel capacity maximization means that each subchannels (therefore, the reciprocal information maximimization problem is given by:

$$C = E_H\left[\max \sum_{i=1}^{r} \vartheta_i \sum_{i=1}^{r} \log_2\left(1+\frac{P_s \vartheta_i}{MN_0}\lambda_i\right)\right] \tag{20}$$

Applying Lagrangian method, the optimal power assigned to each eigenmode is given by:

$$\vartheta_i^{opt} = \left(\mu - \frac{MN_0}{P_s\lambda_i}\right)_+, i = 1,2,\ldots,r \tag{21}$$

$$\sum_{i=1}^{r} \vartheta_i^{opt} = M \tag{22}$$

where μ (Mu) is a constant signifying the water level and the expression:

$$\left(\mu - \frac{MN_0}{P_s\lambda_i}\right)_+ = \begin{cases} x & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases} \tag{23}$$

Now, the optimal power allocation is determined iteratively using the water filling algorithm stated as follows:

The iteration count p is chosen as 1 and then the Mu in (21) calculated using the formula:

$$\mu = \frac{M}{(r-p+1)}\left[1 + \frac{N_0}{P_s}\sum_{i=1}^{r-p+1}\frac{1}{\lambda_i}\right] \tag{24}$$

Employing the Mu value obtained from (24), the assigned power to the ith subchannel can be found from the expression given by:

$$\vartheta_i = \left(\mu - \frac{MN_0}{P_s\lambda_i}\right)_+, i = 1,2,\ldots,r - p + 1 \tag{25}$$

If the assigned power to the channel with the lowest gain is negative, that is $MN_0/P_s\lambda_i > \mu$ (meaning that subchannel is bad), such channel is rejected and (21) is set to zero such that the algorithm is rerun with incrementing iteration count by 1. This step is repeated until all good subchannels are assigned the optimal power. Once the allocation of optimal power across the spatial sub-channel is found, the covariance matrix A of the optmized input can now be gotten using:

$$A^{opt} = \text{diag}\left\{\vartheta_1^{opt}, \vartheta_2^{opt}, \cdots, \vartheta_r^{opt}\right\} \tag{26}$$

and the (10) can be reoresented as:

$$C = E_H\left[\log_2 \det\left(I_{N_r} + \frac{P_s}{MN_0} HA^{opt}H^*\right)\right] \tag{27}$$

IV. Simulation Results and Discussion

In this paper, the channel capacity of MU massive MIMO system has been simulated considering different number of BS antennas and number of user terminals over the SNR range from 0 to 25. The number of BS antennas is: M = 300, 400, 500, 600, while the number of receive antennas which is equal to the number of UE is: N = 50, 200.

The simulation results of the channel capacity of the proposed massive multiple antenna system are presented for different antenna configurations over the signal to noise ratio (SNR) range from 0 to 25 dB. Simulations were carried out by considering two scenarios: when the channel is known (tagged by the legend with) and unknown (tagged by the legend without) to the transmitter as well as the capacity difference. The simulation study considered the cases: when the number of base station antennas (transmit antennas), M = 300, 400, 500, 600 and the number of receive antennas, N = 50, 200 in Fig. 3 and 4. Table 1 presents the performance analyses of the results.

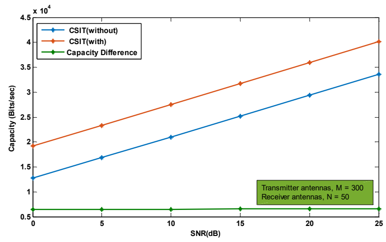

Fig. 3a Capacity variation with SNR for M = 300 and N = 50

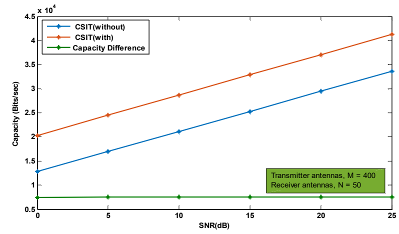

Fig. 3b Capacity variation with SNR for M = 400 and N = 50

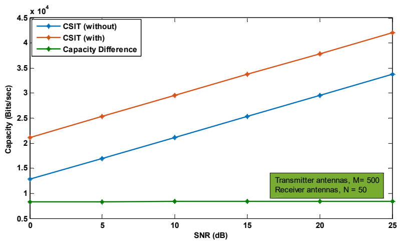

Fig. 3c Capacity variation with SNR for M = 500 and N = 50

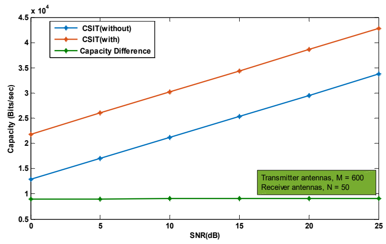

Fig. 3d Capacity variation with SNR for M = 600 and N = 50

Fig. 3 Capacity variation with SNR (M = 300,400,500,600, N = 50)

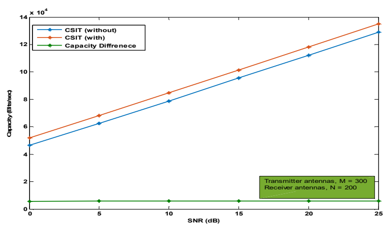

Fig. 4a Capacity variation with SNR for M = 300 and N = 200

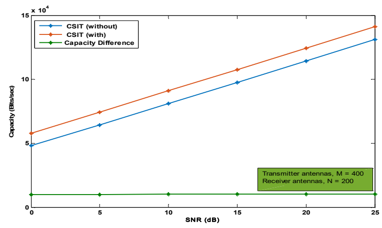

Fig. 4b Capacity variation with SNR for M = 400 and N = 200

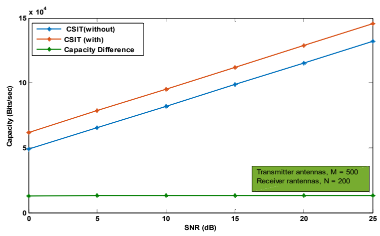

Fig. 4c Capacity variation with SNR for M = 500 and N = 200

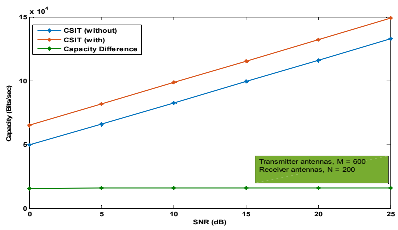

Fig. 4d Capacity variation with SNR for M = 600 and N = 200

Fig.4 Capacity variation with SNR (M = 300,400,500,600, N = 200)

Table 1 Analysis Of System Capacity (M = 300,400,500,600, N = 50, 200, At Snr = 25 Db)

Transmit Antennas (M)	**Receive Antennas (N)**					
	50			**200**		
	Unkonwn Channel (Bits/sec)	**Known Channel (Bits/sec)**	**Capacity Difference (Bits/sec)**	**Unkonwn Channel (Bits/sec)**	**Known Channel (Bits/sec)**	**Capacity Difference (Bits/sec)**
300	3.358e+04	4.011e+04	6627	1.291e+05	1.35e+05	5906
400	3.367e+04	4.124e+04	7575	1.131e+05	1.412e+05	1.01e+04
500	3.372e+04	4.211e+04	8387	1.329e+05	1.489e+05	1.601e+04
600	3.376e+04	4.281e+04	9052	1.322e+05	1.456e+05	1.355e+04

It can be deduced in Table 1 that the channel capacity is higher when the channel state information (CSI) is known to the transmitter than when it is unknown. This is so because for known state, the transmitter sends through the sub-channels that are good but when channel is not known, the transmitter divides the power evenly among all the sub-channels without considering the state of each sub-channel (whether good or bad). Therefore, power transmitted through bad sub-channels is wasted and does not account to total channel capacity. It can also be seen that when the number of transmit antennas gets higher, the the capacity gained increases. Also, as the number of antennas at the receive side increases with respect to M as shown in the Table, the capacity of the channel is improved. For instance, N = 200 yield better channel capacity than N = 50 when M = 600. Generally, increase in the base station antennas (transmit antennas) M can improve channel capacity as shown in Table 1 where the highest capacity (4.281e+04 Bits/sec, 1.456e+05 Bits/sec) is achieved at M = 600, and for K = 50, 200 respectively.

V. CONCLUSION

This paper has studied the capacity of MU massive MIMO downlink channel at different antenna configurations.Using water filling algorithm developed with MATLAB codes, the channel capacity in terms of channel state information (CSI) performance of massive MIMO system with up to 600 base station antennas and 200 user equipments (UEs) has been examined. Making the number of antennas at the transmit side larger than the number of receive antennas with known channel state information (CSI) increases system channel capacity.

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