Find A General Solution Of An Equation Of The Hyperbolic Type With A Second-Order Singular Coefficient And Solve The Cauchy Problem Posed For This Equation

Karimova Shalola Musayevna¹ and Melikuzieva Dilshoda Mukhtorjon qizi²

¹Namangan Engineering Construction Institute, Namangan, Republic of Uzbekistan
²Namangan Engineering Construction Institute, Namangan, Republic of Uzbekistan

Abstract – This paper presents a general solution of a hyperbolic type equation with a second-order singular coefficient and a solution to the Cauchy problem posed for this equation.

Keywords – Singular, coefficient, characteristic equation, canonical equation, Cauchy problem.

Consider an equation of the hyperbolic type with a second-order singular coefficient,

$$-(y)^{m} u_{xx} + u_{yy} + \frac{\beta}{y} u_y = 0 \quad (1)$$

$$m > 0, \ y < 0, \ - \frac{m}{2} \leq \beta_0 < 1$$

To make the equation look canonical, we find its characteristics,

$$-(-y)^{m} dy^2 + dx^2 = 0$$

$$\left(\frac{dx}{-(-y)^{m}} \ dy\right) \cdot \left(\frac{dx}{-(-y)^{m}} \ dy\right) = 0$$

$$\xi = x + \frac{2}{m+2} (-y)^{\frac{m+2}{2}} \ \text{and} \ \eta = x - \frac{2}{m+2} (-y)^{\frac{m+2}{2}} \quad (2)$$

New $\xi$ and $\eta$ We calculate the products involved in equation (1) by entering the variables:
Find a general solution of an equation of the hyperbolic type with a second-order singular coefficient and solve the Cauchy problem posed for this equation.

\[ u_{xx} = v_{\xi \xi} + 2v_{\xi \eta} + v_{\eta \eta} \]

\[ u_{\eta} = -(y)^{m/2} v_{\xi} + (y)^{m/2} v_{\eta} \]

\[ u_{\eta \eta} = (y)^{m/2} v_{\xi \xi} + (y)^{m/2} v_{\eta \eta} - 2v_{\xi \eta} (y)^{m/2} \frac{m-2}{2} v_{\xi} - \frac{m}{2} (y)^{m/2} v_{\eta} = 0 \]

Substituting the findings into Equation (1), we obtain the following canonical equation.

\[ v_{\eta \eta} (y)^{m/2} \left[ \frac{m+2 \beta_0}{8} \right] v_{\xi} + (y)^{m/2} \left[ \frac{m+2 \beta_0}{8} \right] v_{\eta} = 0 \]  \[ (1) \]  \[ (3) \]

In Equation (3) we make the following notation,

\[ a = \frac{m+2 \beta_0}{2(m+2)} \], \[ 0 < a < 1 \]  \[ a \xi - \eta = \frac{4}{m+2} (y)^{m/2} \]

After the notation, the appearance of the equation becomes as follows: Euler-Darboux.

\[ v_{\eta \eta} = -a \xi - \eta v_{\xi} + a \xi - \eta v_{\eta} = 0 \]

\[ u_{\xi \eta} = -(y)^{m/2} \left[ \frac{m+2 \beta_0}{8} \right] u_{\xi} + (y)^{m/2} \left[ \frac{m+2 \beta_0}{8} \right] u_{\eta} = 0 \]

\[ u_1(x,y) = \int_{x}^{y} \Phi(\xi) (\xi - x)^{-\alpha} \cdot (y - \xi)^{-\beta} \, d\xi \]

\[ u_2(x,y) = (y-x)^{1-2a} \int_{x}^{y} \Psi(\xi) (\xi - x)^{a-1} (y - \xi)^{a-1} \, d\xi \]

\[ \Phi(\xi) \] and \[ \Psi(\xi) \] - is an arbitrary continuous function that satisfies equation (5).

\[ \xi = x(1-t) + yt \]

If we make the substitution, the general solution of equation (4) can be found in the following form:

\[ v(\xi, \eta) = \left( \eta - \xi \right)^{1-2a} \int_{0}^{1} \Phi \left[ \xi + t(\eta - \xi) \right] \cdot t^{a-1} \cdot (1-t)^{a-1} \, dt \]

\[ + \int_{0}^{1} \Psi \left[ \xi + t(\eta - \xi) \right] \cdot t^{a-1} \cdot (1-t)^{a-1} \, dt \]

\[ \Phi(\xi) \] and \[ \Psi(\xi) \] Substituting Equation (2) for

\[ u(x,y) = \left[ - \frac{4}{m+2} (y)^{m/2} \right] \int_{0}^{1} \Phi \left[ x + (1-2t) \frac{2}{m+2} (y)^{m/2} \right] \cdot t^{a-1} \cdot (1-t)^{a-1} \, dt \]

\[ + \int_{0}^{1} \Psi \left[ x + (1-2t) \frac{2}{m+2} (y)^{m/2} \right] \cdot t^{a-1} \cdot (1-t)^{a-1} \, dt \]

\[ \Phi(\xi) \] and \[ \Psi(\xi) \] Substituting Equation (2) for

\[ \lim_{y \to 0} u(x,y) = \tau(x) \]

\[ \lim_{y \to 0} (-y)^{\beta_0} u_{\xi}(x,y) = \nu(x) \]
Find a solution that satisfies the conditions. Here $\tau(x)$ and $\nu(x)$ - given features,

$\tau(x) \in C[-1;1] \cap C^2 (-1;1)$

$\nu(x) \in C^2 (-1;1)$ belongs to the class.

$\nu(x)$ function $x \rightarrow \pm 1$ can suddenly achieve a small specificity.

Substituting Equation (9) into Condition (10), we obtain

$$\Psi(x) \cdot \int_0^1 t^{a-1} \cdot (1-t)^{a-1} \, dt = \tau(x) \quad (12)$$

$$\psi(x) = \frac{\Gamma(2a) \cdot \tau(x)}{\Gamma^2 (a)} \quad (13)$$

If we set equation (9) to condition (11),

$$\Phi(x) = \left[ \frac{4}{m+2} \frac{2\beta-2}{\beta+1} \frac{\Gamma(2-2a) \cdot \nu(x)}{\Gamma^2 (1-a)} \right] \quad (14)$$

we have equality.

If we reduce the findings to equation (9), we get the solution of the Cauchy problem for equation (1),

$$u(x, y) = 2 \frac{1}{\beta_0-1} \frac{\Gamma(2-2a)}{\Gamma^2 (1-a)} (-y)^{\beta_0}$$

$$\int_0^1 \left[ x + \frac{2}{m+2} \left( -y \right)^{\frac{m+2}{2}} \left( 2t-1 \right) \right] \cdot t^{a-1} \cdot (1-t)^{a-1} \, dt$$

$$+ \frac{\Gamma(2a)}{\Gamma^2 (a)} \int_0^1 \left[ x + \frac{2}{m+2} \left( -y \right)^{\frac{m+2}{2}} \left( 2t-1 \right) \right] \cdot t^{a-1} \cdot (1-t)^{a-1} \, dt \quad (15)$$

REFERENCES

